Fast Wake Algorithm Derivation

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The fundamental linearized fluid dynamic equations are

$$\frac{\partial h(\vec{\mathbf{x}},t)}{\partial t} = \left(-\partial^2\right)^{1/2} \phi(\vec{\mathbf{x}},t) \tag{1}$$

$$\frac{\partial \phi(\vec{\mathbf{x}},t)}{\partial t} = -gh(\vec{\mathbf{x}},t) + S(\vec{\mathbf{x}} - \vec{\mathbf{u}}t) \tag{2}$$

with the definitions:

- h is the waveheight,
- ϕ is the velocity potential,
- $\vec{\mathbf{x}}$ is the postion on the water surface,
- t is time,
- S is the shape of the wake source,
- $\vec{\mathbf{u}}$ is the source velocity,
- g is the gravitational constant

We solve this by taking the FFT in space and time, to get

$$\tilde{h}(\vec{\mathbf{k}},\omega) = \frac{1}{g}\frac{\tilde{S}(\vec{\mathbf{k}})}{1-\omega^2/gk}\delta(\omega - \vec{\mathbf{u}}\cdot\vec{\mathbf{k}}) \tag{3}$$

Now doing the temporal frequecy integral, we have

$$\tilde{h}(\vec{\mathbf{k}},t) = \exp(-i\vec{\mathbf{k}}\cdot\vec{\mathbf{u}}t)\,\frac{1}{g}\frac{\tilde{S}(\vec{\mathbf{k}})}{1-(\vec{\mathbf{u}}\cdot\vec{\mathbf{k}})^2/gk} \tag{4}$$

Now all that needs to be done is build the source shape in Fourier space, and apply it here in the inverse fft. Just be carefull of the pole.