



VISUALIZATION OF SIMULATED OCEAN SCENES EMPLOYING WHITECAP FRACTION PHENOMENOLOGY

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Overview

- Constructing & simulating ocean surfaces
 - Light Transport Rendering via Monte Carlo
 - Representating whitecaps using a threshold parameter
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- Threshold selection using statistical models of whitecap fraction
 - Rendered ocean with whitecaps
 - Conclusions

Part I

Part II

Constructing & Simulating Ocean Surfaces

Christopher J. Horvath. 2015. "Empirical Directional Wave Spectra for Computer Graphics." 2015 Symposium on Digital Production (DigiPro '15). ACM, New York, NY, USA, 29–39
J. Tessendorf, 2004, "Simulating Ocean Water", https://people.cs.clemson.edu/~jtessen/reports/papers_files/coursenotes2004.pdf

- Ocean surface height displacement is a random realization of a spectrum in Fourier space, evolving via linearized Bernoulli equation with a dispersion relationship

$$h(\mathbf{x}) = \int \frac{d^2 k}{2\pi^2} \left\{ h_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t} + h_{-\mathbf{k}}^* e^{-i\omega_{-\mathbf{k}}^* t} \right\} e^{i\mathbf{k} \cdot \mathbf{x}}$$
$$\langle h_{\mathbf{k}} \rangle = 0$$
$$\langle |h_{\mathbf{k}}|^2 \rangle \propto P(\mathbf{k})$$
$$\omega_{\mathbf{k}}^2 = g|\mathbf{k}| \tanh(|\mathbf{k}|d)$$

- Fourier transform implemented numerically using Fast Fourier transform on a patch of ocean

$$h(\mathbf{x}) = FFT \left\{ h_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t} + h_{-\mathbf{k}}^* e^{-i\omega_{-\mathbf{k}}^* t} \right\}$$

- Periodicity of FFT provides arbitrary tiling over vast regions, with repetitive features.
- Linearly adding multiple height fields suppresses repetition with more control over simulation of the maritime environment.

$$h(\mathbf{x}, t) = \sum_i h_i(\mathbf{x}, t)$$



Horizontal Displacement

J. Tessendorf, 2004, "Simulating Ocean Water", https://people.cs.clemson.edu/~jtessen/reports/papers_files/coursenotes2004.pdf

- Gerstner-wave-like horizontal displacement derives from velocity potential $\phi(\mathbf{x}, t)$

$$\mathbf{V}(\mathbf{x}, t) = \nabla_{\mathbf{x}} \phi(\mathbf{x}, t) \quad \mathbf{D}(\mathbf{x}, t) = \int_0^t dt' \mathbf{V}(\mathbf{x}, t')$$

$$\mathbf{X}(\mathbf{x}, t) = \mathbf{x} + \mathbf{D}(\mathbf{x}, t)$$

$$\mathbf{D}(\mathbf{x}) = f_{cusp} \text{ FFT} \left\{ i \frac{\mathbf{k}}{|\mathbf{k}|} \left(h_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t} + h_{-\mathbf{k}}^* e^{-i\omega_{-\mathbf{k}}^* t} \right) \right\}$$

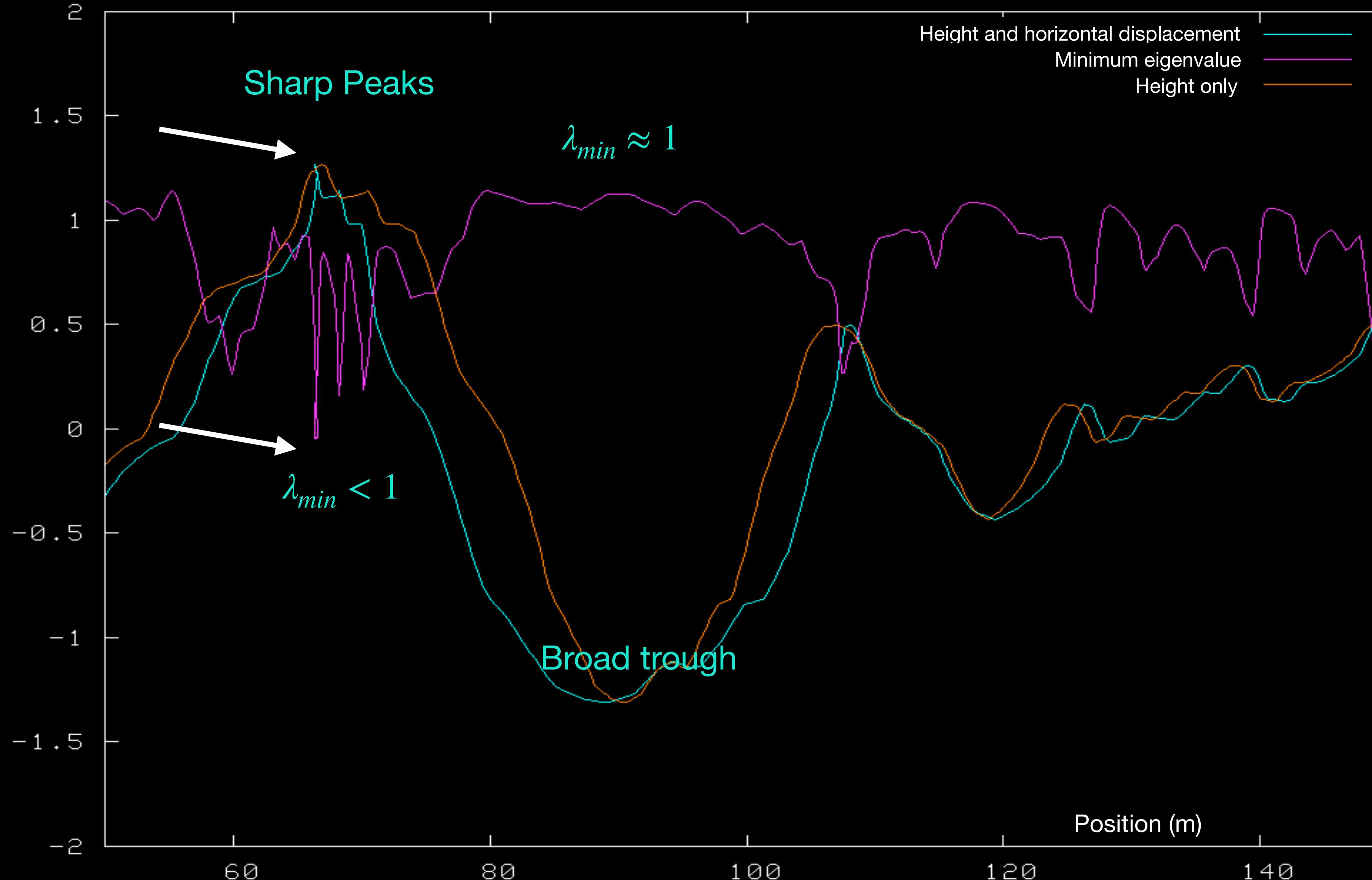
- Horizontal displacement alters surface area via Jacobian with eigenvalues

$$\nabla_{\mathbf{x}} \mathbf{X}(\mathbf{x}, t) = \begin{bmatrix} 1 + \frac{\partial D_x}{\partial x} & \frac{\partial D_x}{\partial z} \\ \frac{\partial D_z}{\partial x} & 1 + \frac{\partial D_z}{\partial z} \end{bmatrix} \quad \det(\nabla_{\mathbf{x}} \mathbf{X}(\mathbf{x}, t)) = \lambda_{max} \lambda_{min}$$

- Minimum eigenvalue is a sensitive measure of steep peaks
 - No displacement: $\lambda_{min} = \lambda_{max} = 1$
 - Near sharp peaks: $\lambda_{min} \rightarrow 0$
 - Broad troughs: $\lambda_{min} \geq 1$



Minimum Eigenvalue and Wave Height



Light Transport Rendering via Monte Carlo

Pharr, Jacob, Humphreys, 2018, PHYSICALLY BASED RENDERING, <https://pbr-book.org>

Dutre, Bekaert, Bala, 2003, ADVANCED GLOBAL ILLUMINATION, A.K. Peters Ltd.

- Well-established theory of Global Illumination, derived from radiative transfer for surfaces
- Integral equation relating incoming, emitted, and outgoing radiance at each point and direction on each surface

$$L_O(\mathbf{p}, \hat{\mathbf{n}}) = \int d\Omega' B(\mathbf{p}, \hat{\mathbf{n}}, \hat{\mathbf{n}}') |\hat{\mathbf{n}}_S \cdot \hat{\mathbf{n}}'| L_I(\mathbf{p}, \hat{\mathbf{n}}') + L_E(\mathbf{p}, \hat{\mathbf{n}})$$

- Implemented using ray path tracing and Monte Carlo reflection, refraction & emission of rays to/from surfaces.
- Normal calculations at full simulation resolution (“Normal Mapping for Rendering Vast Oceans”, paper in progress)
- Fresnel reflection & refraction
- Ocean surface tessellated into triangles with increasing size further from camera.
- Hyperspectral channels, polarization, and volume scatter possible

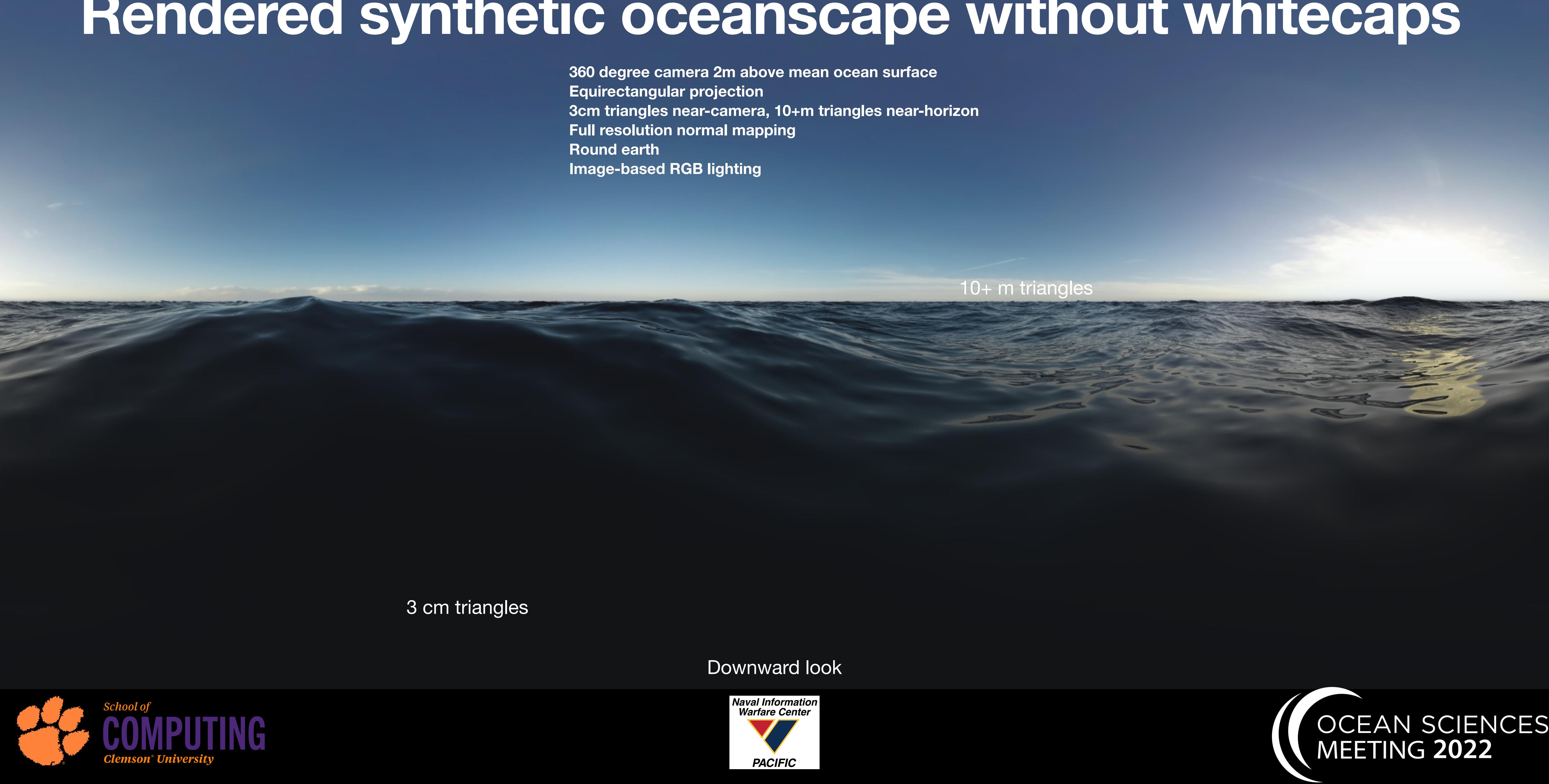
Upward look

Rendered synthetic oceanscape without whitecaps

360 degree camera 2m above mean ocean surface
Equirectangular projection
3cm triangles near-camera, 10+m triangles near-horizon
Full resolution normal mapping
Round earth
Image-based RGB lighting



10+ m triangles



3 cm triangles

Downward look



Representing Whitecaps using a Threshold Parameter

Egbert S. Tse, John McGill, and Robert L. Kelly. 1990. Coherent whitecap and glitter simulation model. In Ocean Optics X, Richard W. Spinrad (Ed.), Vol.1302. International Society for Optics and Photonics, SPIE, 505–519. <https://doi.org/10.1117/12.21468>

- Dynamically updated map O of whitecap content with growth and decay
- Decay mechanism: exponential damping with half-life T
- Growth mechanism: map value boosted where & when minimum eigenvalue crosses threshold
- “Dynamical Update” from time t to next:

$$O(\mathbf{x}, t + \Delta t) = \begin{cases} 1 & \lambda_{min}(\mathbf{x}, t + \Delta t) < \lambda_T \quad (\text{Growth}) \\ O(\mathbf{x}, t) e^{-\Delta t/T} & \lambda_{min}(\mathbf{x}, t + \Delta t) \geq \lambda_T \quad (\text{Decay}) \end{cases}$$

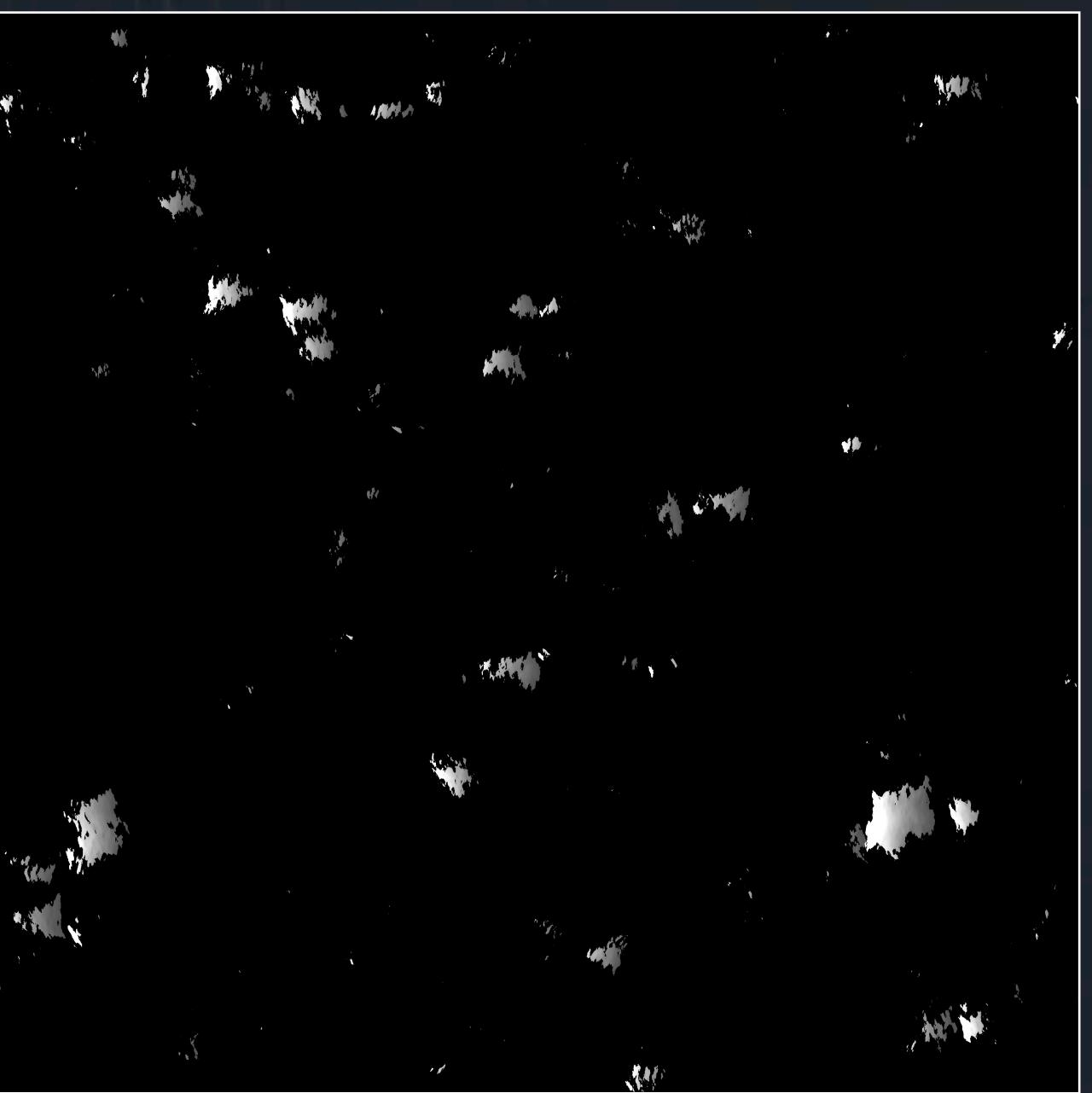
$$O(\mathbf{x}, t + \Delta t) = O(\mathbf{x}, t) e^{-\Delta t/T} + \Theta(\lambda_T - \lambda_{min}(\mathbf{x}, t + \Delta t)) \left(1 - O(\mathbf{x}, t) e^{-\Delta t/T}\right)$$

Heaviside Step Function $\Theta(q) = \begin{cases} 1 & q > 0 \\ 0 & q < 0 \end{cases}$

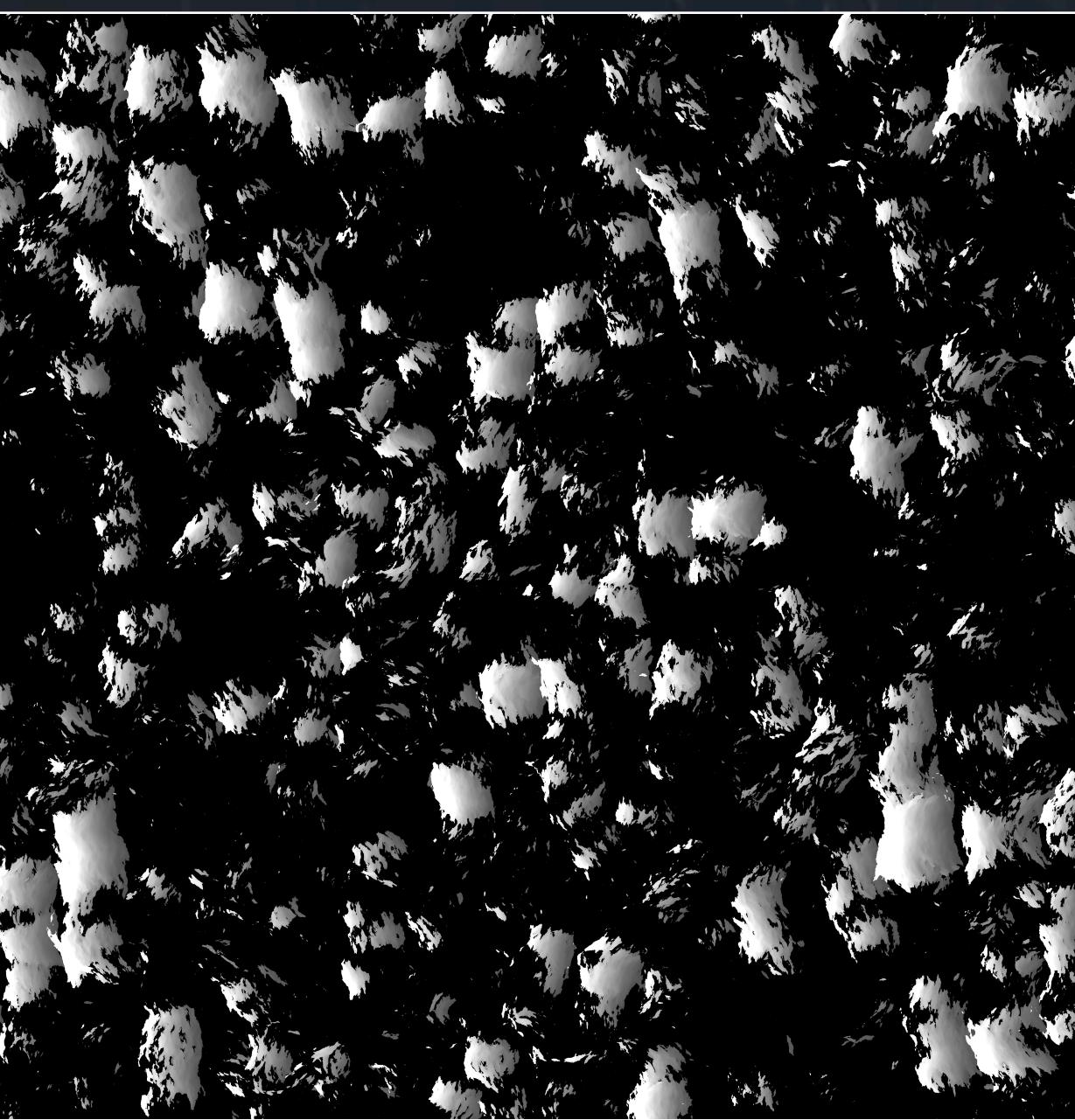


Maps from Arbitrary Minimum Eigenvalue Thresholds

Whitecap fraction W = average of map value over the domain of the map



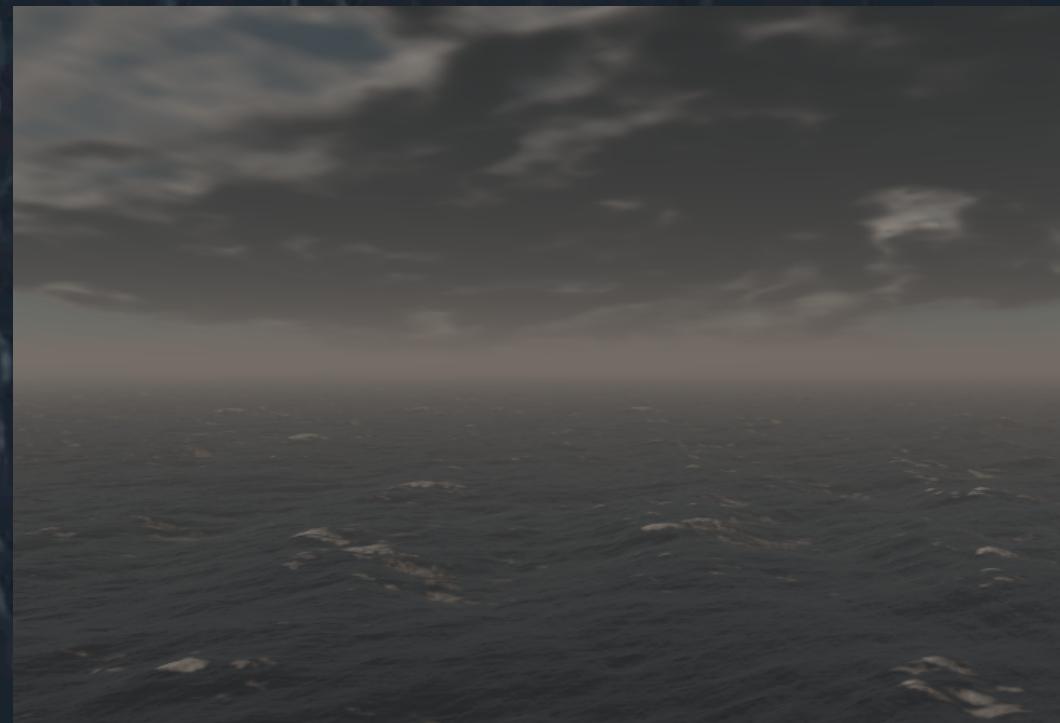
$$W = 0.011$$
$$\lambda_T = 0.71$$



$$W = 0.15$$
$$\lambda_T = 0.81$$

Previous work

Minimum eigenvalue threshold
(With time history)

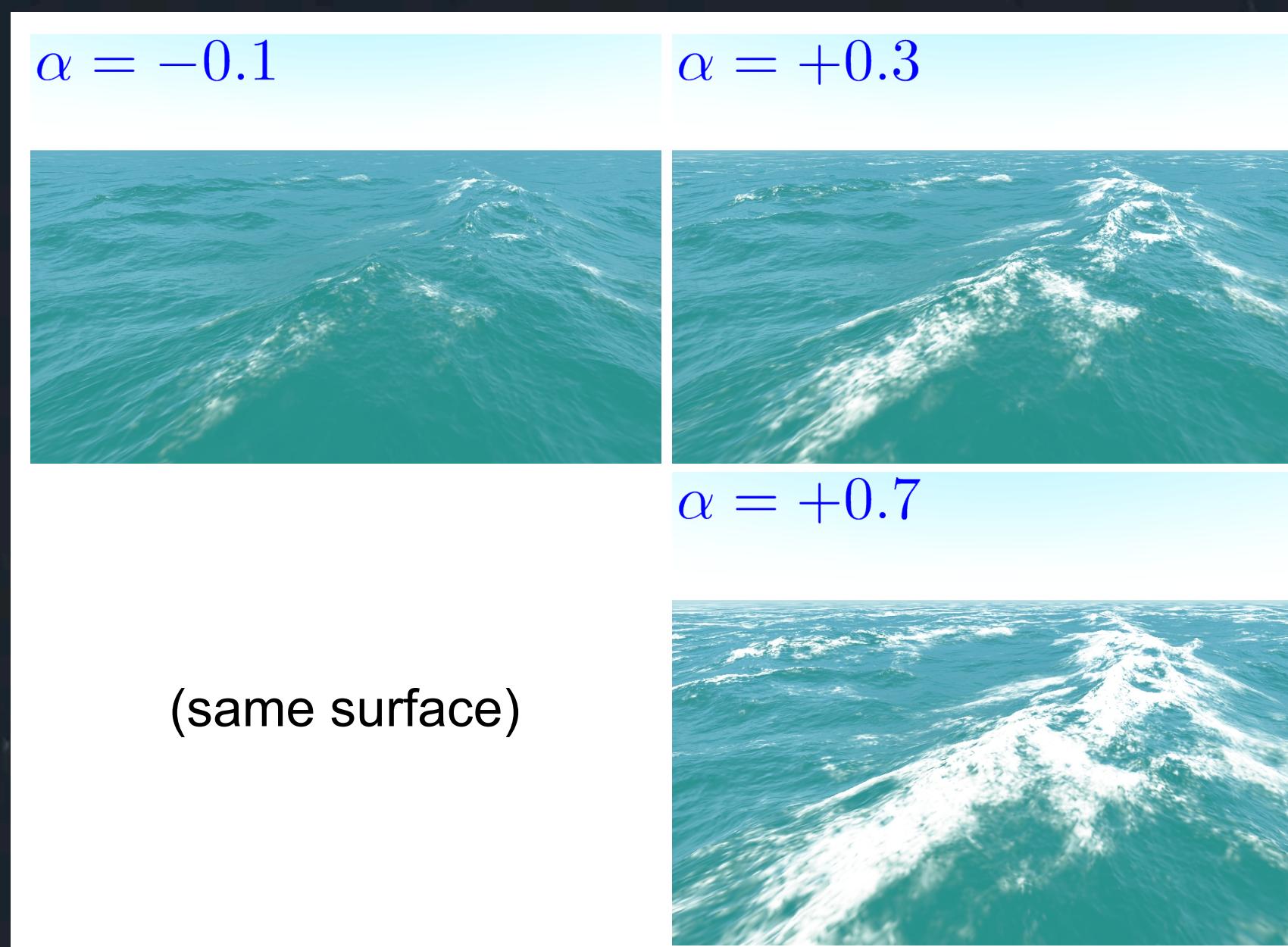


RenderWorld Scene Simulator (1996)



SUPERMAN RETURNS R&D clip (2005)

Jacobian-determinant-based shader (2012)
(Does not include grow+decay)

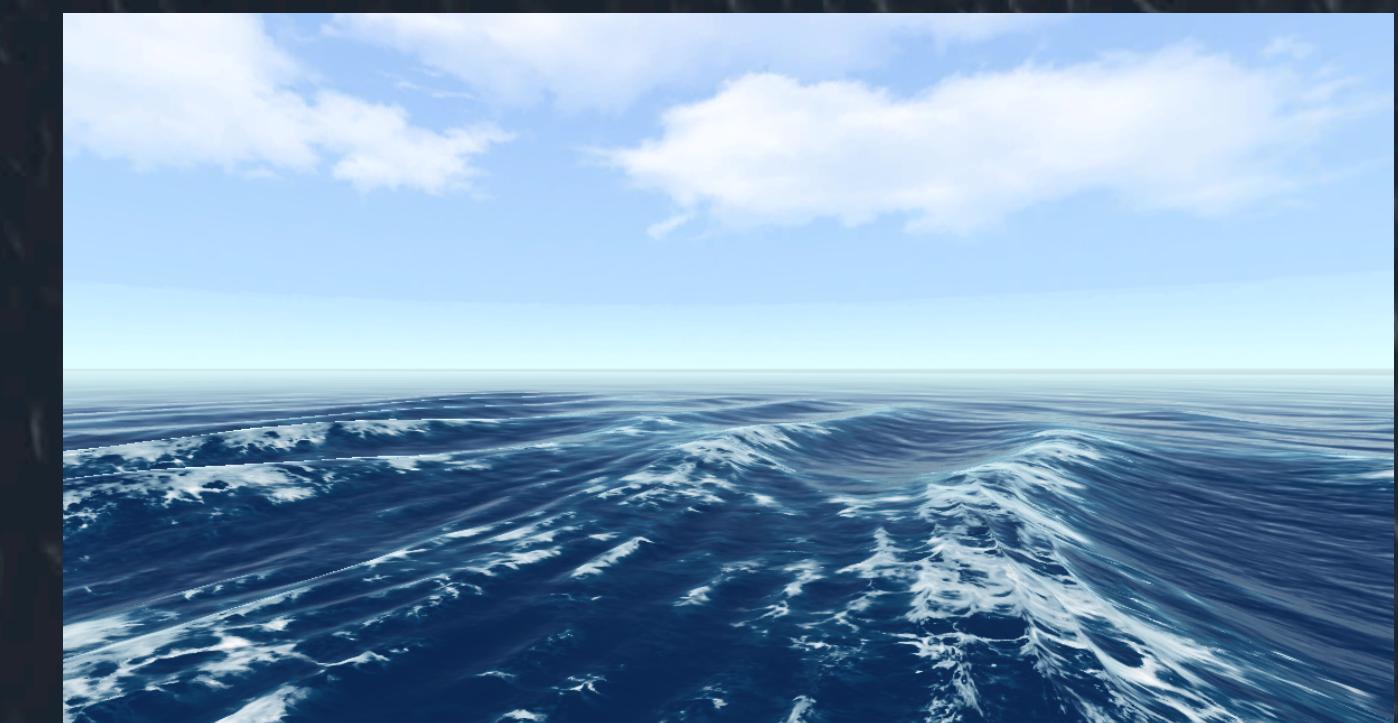


Real-time Animation and
Rendering of Ocean Whitecaps

Jonathan Dupuy, Eric Bruneton

<https://hal.inria.fr/hal-00967078/file/Whitecaps-presentation.pdf>

Jacobian-determinant-based game tools (2017)
(Includes grow+decayy)



Crest: Novel ocean rendering techniques
in an open source framework

Huw Bowles,
Daniel Zimmermann,
Chino Noris,
Beibei Wang

<https://advances.realtimerendering.com/s2017/index.html>



Threshold Selection via Statistical Models of Whitecap Fraction

- Oceanographic studies and models relate average whitecap fraction W to spectral parameters like wind speed U .
M. F. M. A. Albert, M. D. Anguelova, A. M. M. Manders, M. Schaap, and G. de Leeuw. 2016. Parameterization of oceanic whitecap fraction based on satellite observations. *Atmospheric Chemistry and Physics* 16, 21 (2016), 13725–13751. <https://doi.org/10.5194/acp-16-13725-2016>

$$\star \quad W = 3.84 \times 10^{-6} U^{3.41} \quad (\text{MOM 83})$$

- Assuming statistical stationarity and homogeneity, $W = \langle O(\mathbf{x}, t) \rangle$
- Averaging dynamical update model relates W to the threshold cumulative distribution function (CDF), assuming statistical de-correlation of minimum eigenvalue fluctuations from whitecap map fluctuations

$$W = W\alpha + \text{CDF}(\lambda_T) (1 - W\alpha)$$

$$\star \quad \text{CDF}(\lambda_T) = \langle \Theta(\lambda_T - \lambda_{min}) \rangle = \int_0^{\lambda_T} d\lambda P(\lambda)$$

- Model allows solving for threshold value to achieve selected W on average.

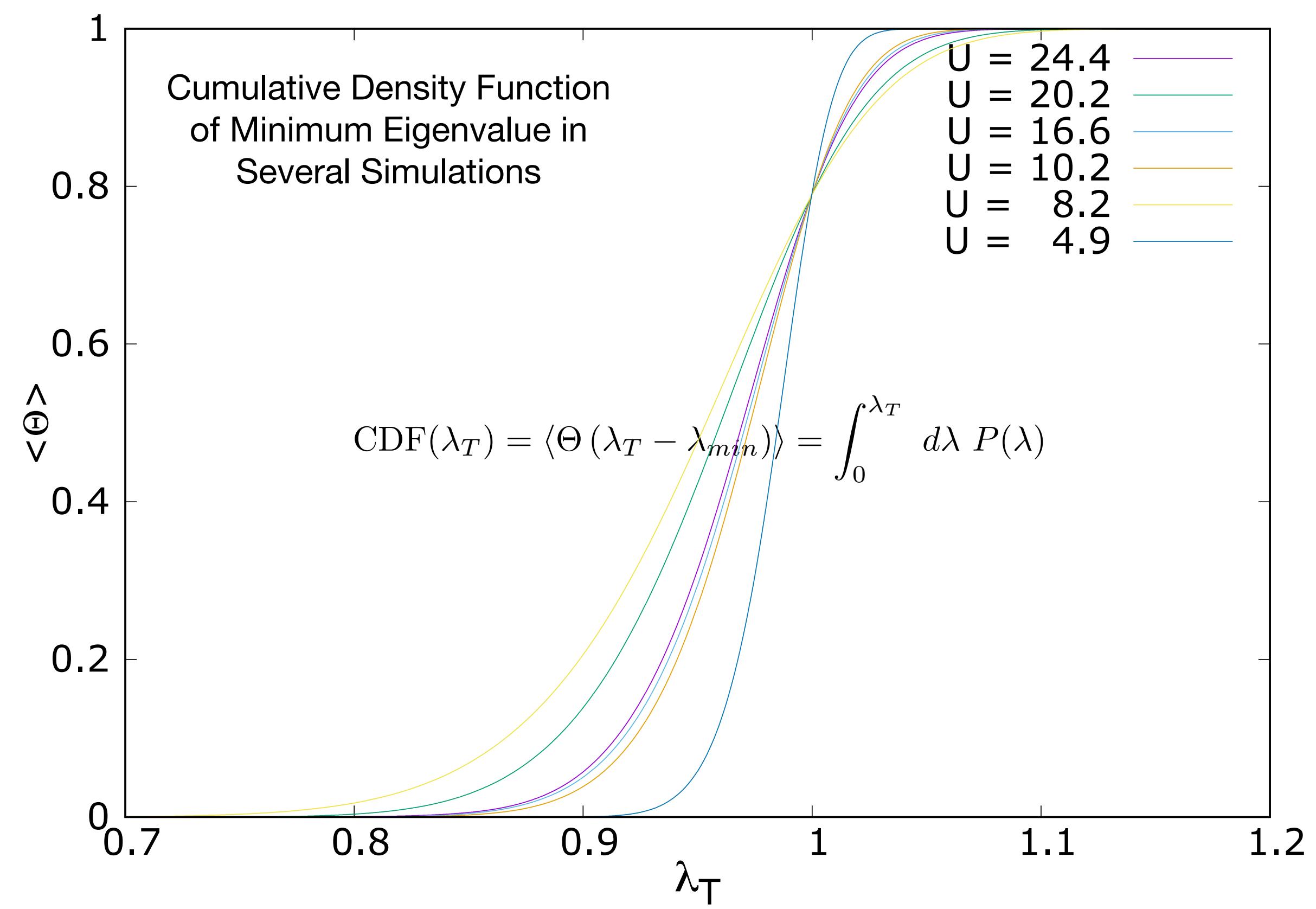
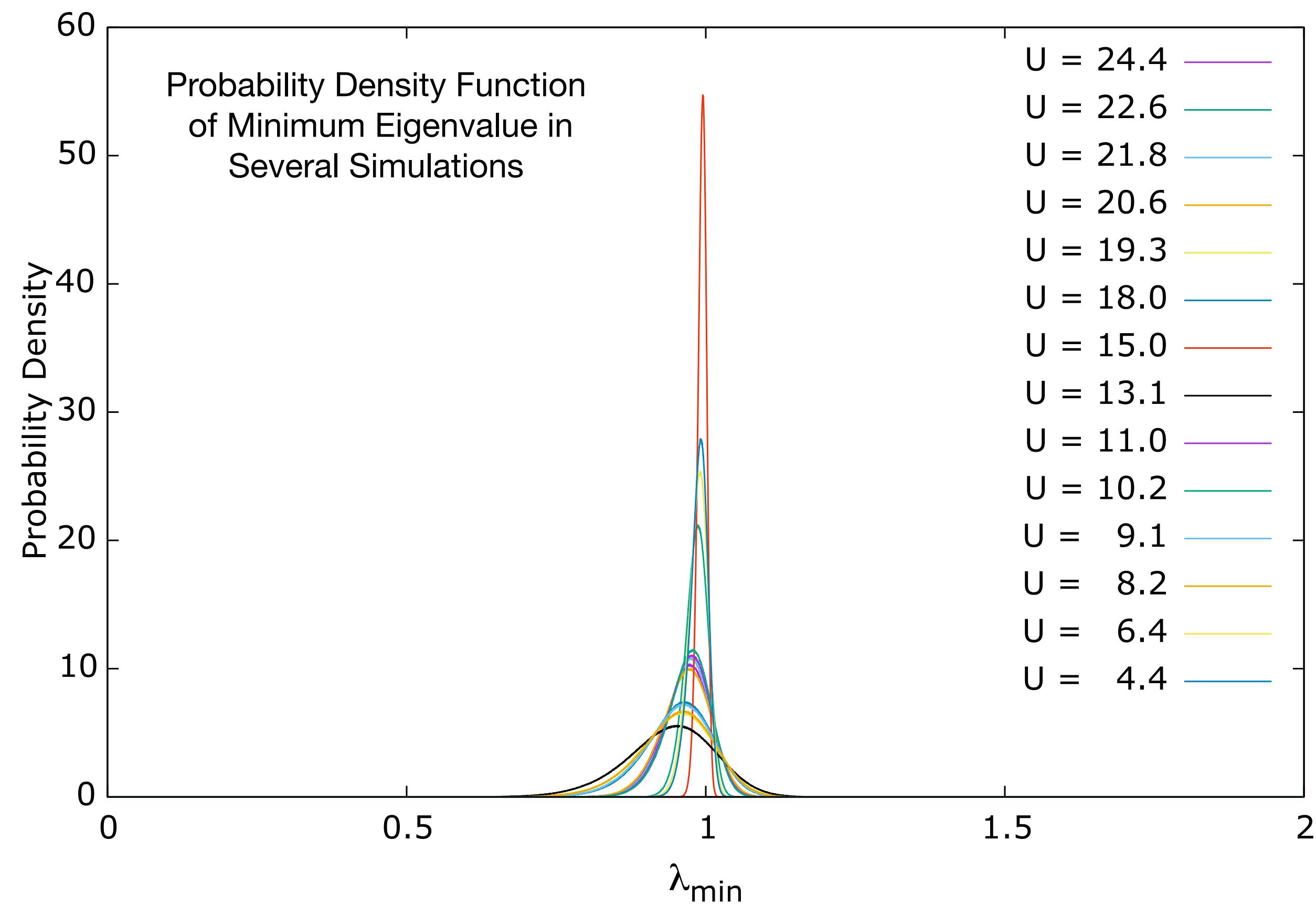
$$\star \quad \frac{W(1 - \alpha)}{1 - W\alpha} = \text{CDF}(\lambda_T) \quad \star \quad \alpha = \exp(-\Delta t/T)$$

- Strategy:
 1. From simulated frame(s), extract minimum eigenvalue probability density function (PDF) via histogram of values.
 2. Integrate PDF it into the CDF \star
 3. For desired W , \star determine needed CDF value. \star
 4. For derived CDF value, extract threshold of minimum eigenvalue. This determines a value for λ_T \star



PDF and CDF of Simulated Ocean Surfaces

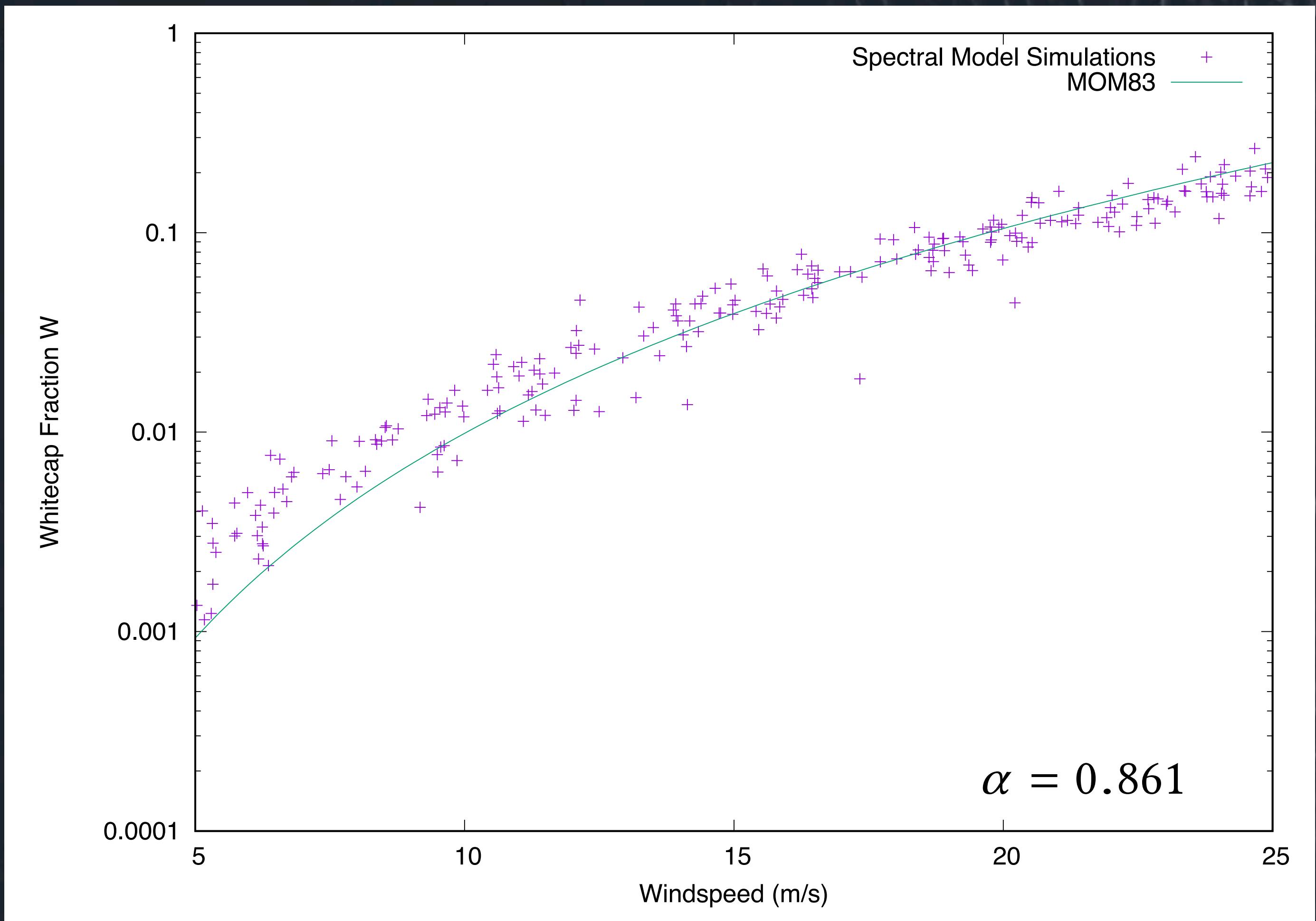
Minimum Eigenvalue statistics

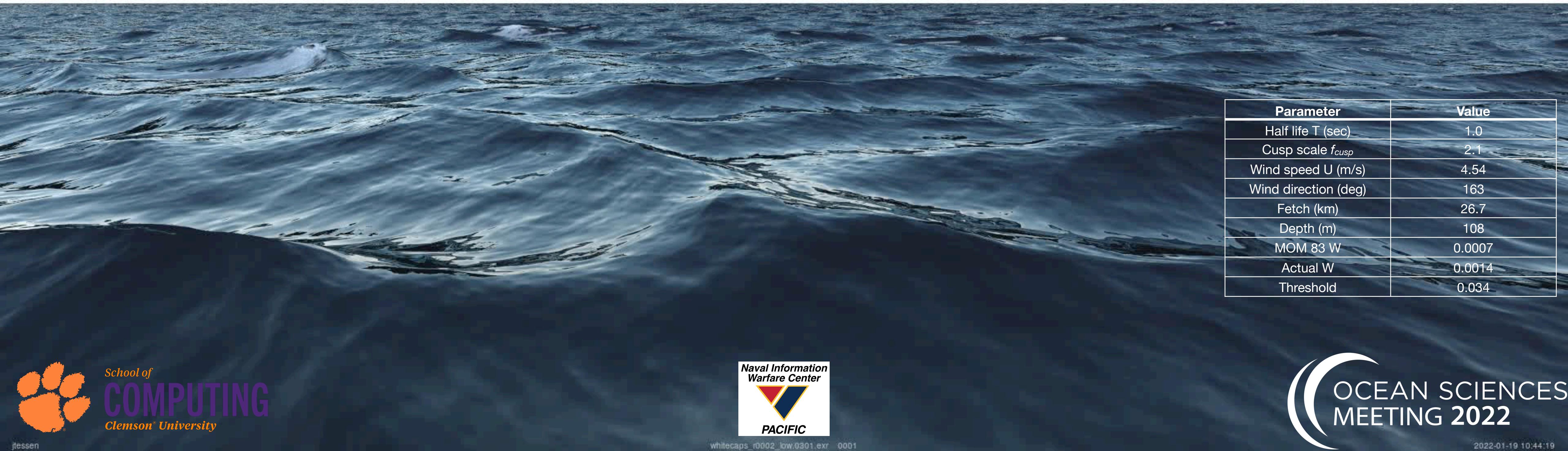


Simulated Whitecap Fraction vs Model

- MOM 83 whitecap fraction model
- 105 simulation runs using TMA spectrum
- 10 second run-up. W averaged over last 3.3 seconds

Parameter	Range
Half life T (sec)	{ 1, 8 }
Cusp scale f_{cusp}	{ 0.01, 0.95 }
Wind speed U (m/s)	{ 4, 25 }
Wind direction (deg)	{ 0, 359 }
Fetch (km)	{ 25, 200 }
Patch size (m)	{400,4000}
Patch dimensions (pixels)	2048 X 2048
Bottom depth (m)	{100,1000000}



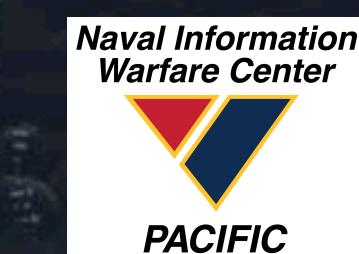


Parameter	Value
Half life T (sec)	1.0
Cusp scale f_{cusp}	2.1
Wind speed U (m/s)	4.54
Wind direction (deg)	163
Fetch (km)	26.7
Depth (m)	108
MOM 83 W	0.0007
Actual W	0.0014
Threshold	0.034

Nadir look
Camera altitude: 600m
FOV: 90 deg



Grazing look
Camera altitude: 5m
FOV: 90 deg



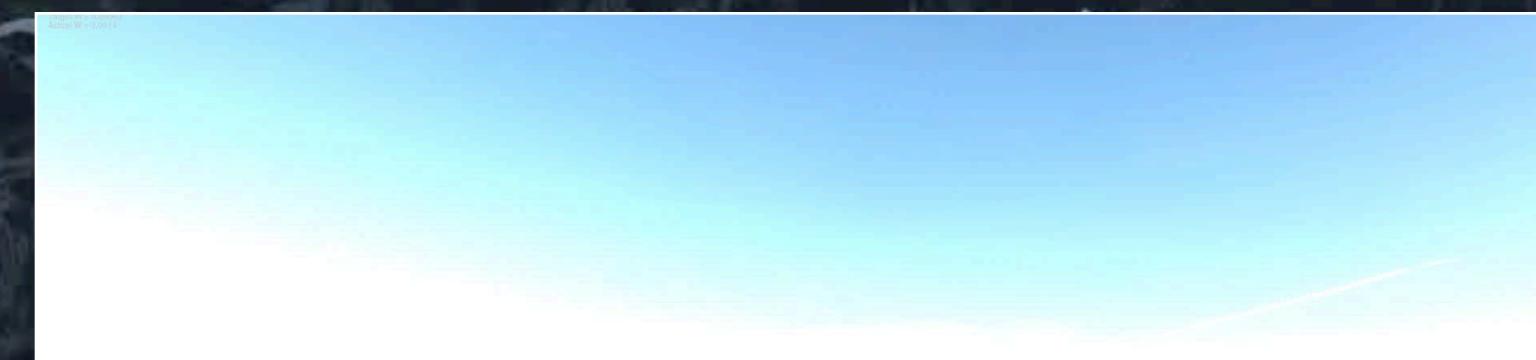
Parameter	Value
Half life T (sec)	1.0
Cusp scale f_{cusp}	2.1
Wind speed U (m/s)	4.54
Wind direction (deg)	163
Fetch (km)	26.7
Depth (m)	108
MOM 83 W	0.0007
Actual W	0.0014
Threshold	0.034

Target W = 0.00067
Actual W = 0.0014

Nadir look
Camera altitude: 600m
FOV: 90 deg

Grazing camera FOV (90 deg)

Grazing camera position



Grazing look
Camera altitude: 5m
FOV: 90 deg



Parameter	Value
Half life T (sec)	1.0
Cusp scale f_{cusp}	0.597
Wind speed U (m/s)	24.1
Wind direction (deg)	188.5
Fetch (km)	31.4
Depth (m)	1000000
MOM 83 W	0.198
Actual W	0.076
Threshold	0.534



Conclusions

- This approach to threshold parameter selection tracks with assumed whitecap fraction model
- Statistical de-correlation assumption should be evaluated in more detail
- PDF-to-CDF-to-Threshold process is applicable to other choices of physical parameter (e.g. surface acceleration, curvature, jacobian)
- Could be extended to use whitecap fraction models with more dependence one spectral properties.
- Work underway to validate and extend using whitecap measurement sets