Physically Based Effects
Implicit Function Cheat Sheet
CP SC 819
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1 Implicit functions

- Sphere
$$f(\mathbf{x}) = R - |\mathbf{x}| \tag{1}$$

- Torus
$$f(\mathbf{x}) = 4R_{major}^2|\mathbf{x}_\perp|^2 - \left(|\mathbf{x}|^2 + R_{major}^2 - R_{minor}^2\right)^2; \tag{2}$$
where
$$\mathbf{x}_\perp = \mathbf{x} - (\mathbf{x}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}} \tag{3}$$

- Cone
$$f(\mathbf{x}) = \begin{cases} 0 & \mathbf{x} = \mathbf{x}_0 \\ h - (\mathbf{x}-\mathbf{x}_0)\cdot\hat{\mathbf{n}} & (\mathbf{x}-\mathbf{x}_0)\cdot\hat{\mathbf{n}} > h \\ (\mathbf{x}-\mathbf{x}_0)\cdot\hat{\mathbf{n}} & (\mathbf{x}-\mathbf{x}_0)\cdot\hat{\mathbf{n}} < 0 \\ \theta - \cos^{-1}\left(\frac{(\mathbf{x}-\mathbf{x}_0)\cdot\hat{\mathbf{n}}}{|\mathbf{x}-\mathbf{x}_0|}\right) & \text{otherwise} \end{cases} \tag{4}$$

- Plane
$$f(\mathbf{x}) = (\mathbf{x}-\mathbf{x}_0)\cdot\hat{\mathbf{n}}; \tag{5}$$

- Box
$$f(\mathbf{x}) = R - x^p - y^p - z^p; \tag{6}$$

- Icosahedron
$$f(\mathbf{x}) = \begin{cases} -1.8\pi & |\mathbf{x}| > 1.8\pi \\ \cos(x+Ty) + \cos(x-Ty) & |\mathbf{x}| <= 1.8\pi \\ +\cos(y+Tz) + \cos(y-Tz) & \\ +\cos(z-Tx) + \cos(z+Tx) - 2.0 & \end{cases} \tag{7}$$
where $T \approx 1.61803399$ (the golden ratio).

- Steiner Patch
$$f(\mathbf{x}) = -(x^2y^2 + x^2z^2 + y^2z^2 - xyz); \tag{8}$$
- Ellipse
$$f(\mathbf{x}) = 1 - Z^2/R^2_{major} - |\mathbf{x}_\perp|^2/R^2_{minor}; \tag{9}$$
where $Z = \mathbf{x} \cdot \hat{\mathbf{n}}$ and $\mathbf{x}_\perp = \mathbf{x} - Z\hat{\mathbf{n}}$
- Cylinder of height h with axis $\hat{\mathbf{n}}$
Define infinite length cylinder $cyl_\infty(\mathbf{x}) = R - |\mathbf{x}_\perp|$ where $\mathbf{x}_\perp = \mathbf{x} - (\mathbf{x}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}$
Cut intersection to cut off top and bottom
$cyl = cyl_\infty \cap plane(\hat{\mathbf{n}}h/2, -\hat{\mathbf{n}}) \cap plane(-\hat{\mathbf{n}}h/2, \hat{\mathbf{n}})$

2 Constructive Solid Geometry Operations

- Union $h = f \cup g$
$h(\mathbf{x}) = \max(f(\mathbf{x}), g(\mathbf{x}))$
- Intersection $h = f \cap g$
$h(\mathbf{x}) = \min(f(\mathbf{x}), g(\mathbf{x}))$
- Cutout $h = f \wedge g \ \equiv \ f \cap -g$
$h(\mathbf{x}) = \min(f(\mathbf{x}), -g(\mathbf{x}))$

3 Blending

Two implicit functions f and g, blend to
$$h(\mathbf{x}) = 2 - e^{-f(\mathbf{x})} - e^{-g(\mathbf{x})} \tag{10}$$
Many implicit functions $f_i, i = 1, \ldots, N$ blend to
$$h(\mathbf{x}) = N - \sum_{i=1}^{N} e^{-f_i(\mathbf{x})} \tag{11}$$

4 Transforms

- Translate by $\mathbf{x}_t$:
$f_t(\mathbf{x}) = f(\mathbf{x} - \mathbf{x}_t)$
- Scale size of the volume by L:
$f_L(\mathbf{x}) = f(\mathbf{x}/L)$
- Rotate volume by angle θ around axis $\hat{\mathbf{n}}$:
$f_\theta(\mathbf{x}) = f(\mathrm{R}^{-1} \cdot \mathbf{x})$ where R is a rotation matrix:
$\mathrm{R} = \cos(\theta) + \hat{\mathbf{n}} \otimes \hat{\mathbf{n}} \ (1 - \cos(\theta)) + (\sigma_{pauli} \cdot \hat{\mathbf{n}}) \ \sin(\theta)$

5 C++ Class Volume